Environmental Water Quality
BAE 452/552

Session 7

Eutrophication

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Phosphorus Loading Concept

- Applied to lakes for which P is limiting nutrient

1. Vollenweider Loading Plots
2. P Mass balance approach

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Vollenweider Loading Plots

- Vollenweider Loading Plots (1968): plot TP loading (L_P) on areal basis for lakes in north temperate region versus mean depth (H) on log-log paper, and label trophic state:

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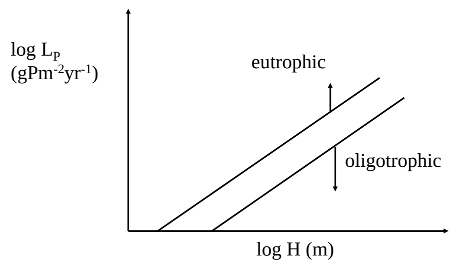
Vollenweider Loading Plots



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Vollenweider Loading Plots

- Then Vollenweider (1975) realized that residence time (τ_w=V/Q) also had an effect: faster flushing caused less eutrophication
- So, plot TP loading (L_P) versus mean depth divided by residence time (H/τ_w) on log-log paper, and label trophic state:

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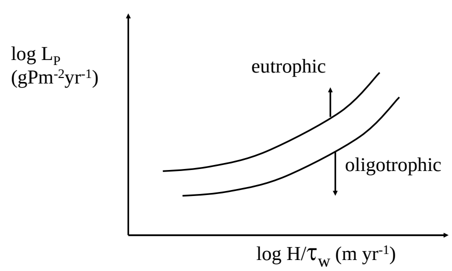
Vollenweider Loading Plots



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Vollenweider Loading Plots

Note that: $\frac{H}{\tau_w} = \frac{HQ}{V} = \frac{HQ}{HA_s} = \frac{Q}{A_s} = q_s$

where

V = Volume (m^3)
Q = discharge ($m^3\ yr^{-1}$)
A_s = surface area (m^2)
q_s = hydraulic overflow rate ($m\ yr^1$)

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Mass Balance

- General mass balance for a well-mixed lake
- Phosphorus mass balance for well-mixed lake

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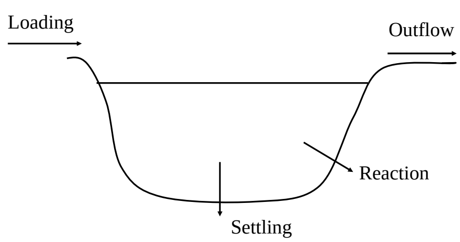
Well-mixed Lake



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Mass Balance for Well-mixed Lake

- Accumulation = Loading – Outflow – Reaction – Settling
- Source: Loading
- Sinks: Outflow, Reaction, Settling

- We need to express each term as a function of measurable variables and parameters

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Accumulation

- Accumulation is the change of mass (M) in the system over time (t): $\text{Accumulation} = \frac{\Delta M}{\Delta t}$
- Mass is related to concentration (c): $M = Vc$

where V is volume (L^{-3})

- If V is constant: $\text{Accumulation} = V\frac{\Delta c}{\Delta t}$
- And for small Δt: $\text{Accumulation} = V\frac{dc}{dt}$

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Loading

- Loading is the sum of loading from all sources: $\text{Loading} = W(t)$

where W(t) is rate of mass loading (MT^{-1})

- As we saw before, loading can also be expressed as: $\text{Loading} = Qc_{in}(t)$

where Q is volumetric flow rate of all water sources entering system (L^3T^{-1}), and $c_{in}(t)$ is average inflow concentration of all sources (ML^{-3})

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Outflow

- Outflow is the rate of mass transport from the system (where $c_{out}=c$) : $\text{Outflow} = Qc$

Reaction

- If we assume a 1st order reaction:

$$\text{Reaction} = kM$$

or

$$\text{Reaction} = kVc$$

where k has the unit T^{-1}

Settling

- Settling losses constitute a mass flux across the surface area of the system: $\text{Settling} = v_s A_s c$

where v_s is apparent settling velocity ($L\ T^{-1}$) representing net effect of settling process, and A_s is the surface area of the lake surface area

$$\text{Loading} = Qc_{in}(t)$$

- Since $V = HA_s$, we can rewrite the equation using a 1st order setting rate $k_s = v_s / H$, and write: $\text{Settling} = k_s V c$

Mass Balance for Well-mixed Lake

The total mass balance:

$$V\frac{dc}{dt} = W(t) - Qc - kVc - v_s A_s c$$

- c is dependent variable,
- t is independent variables
- W(t) is "forcing function"
- V, Q, k, v, and A_s are "parameters"

P Mass Balance for Well-mixed Lake

Assumptions:

- Completely mixed lake (stratification?!)
- Steady state conditions (dynamic behavior?!)
- P limited (ok for NPS)
- TP used as measure of trophic state
- Reaction not significant

P Mass Balance for Well-mixed Lake

The total P mass balance:

$$V\frac{dp}{dt} = W(t) - Qp - v_s A_s p$$

P Mass Balance for Well-mixed Lake

- At steady state: $0 = W(t) - Qp - v_s A_s p$

or $p = \dfrac{W(t)}{Q + v_s A_s}$

Further, if we use areal loading L_p: $L_p = \dfrac{W(t)}{A_s}$ $(g\ P\ m^{-2}\ y^{-1})$

we find: $p = \dfrac{L_p}{q_s + v_s}$

$$L_p = p(q_s + v_s)$$

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P Mass Balance for Well-mixed Lake

Taking logarithms from: $L_p = p(q_s + v_s)$

we find: $\log L_p = \log p + \log(q_s + v_s)$

Slow flushers: $\log L_p = \log p + \log(v_s) = \text{constant}$

Fast flushers: $\log L_p = \log p + \log(q_s)$

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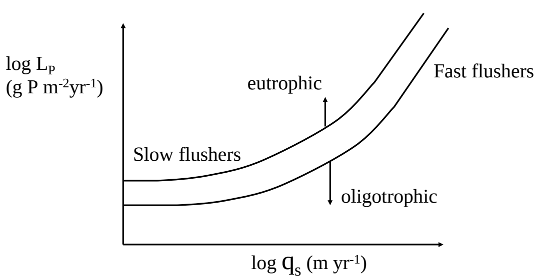
Vollenweider Loading Plots



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P Mass Balance for Well-mixed Lake

$$\log L_p = \log p + \log(q_s + v_s)$$

Difficulty is that v_s is not readily known or can be measured:

Vollenweider used: $v_s = q_s \sqrt{\tau_w} = \dfrac{H}{\sqrt{\tau_w}}$

$$p = \frac{L_p}{q_s + v_s} \longrightarrow p = \frac{L_p}{q_s(1 + \sqrt{\tau_w})}$$

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Flushing Rate

- Flushing rate (ρ) is inverse of residence time:

$$\rho = \frac{1}{\sqrt{\tau_w}} = \frac{Q}{V}$$

$$q_s = H\rho$$

$$v_s = H\sqrt{\rho}$$

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Other Empirical Relationships

Other variables to provide a measure of eutrophication:

1. Phosphorus vs chlorophill-a
2. Chlorophill-a versus Secchi Depth

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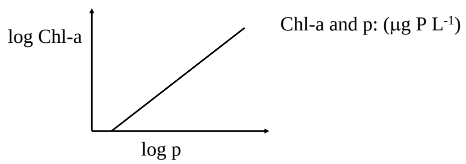
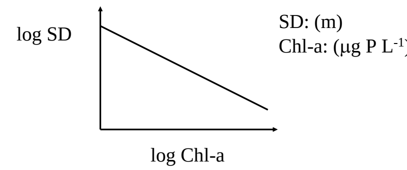
P versus Chlorophill-a

- Log (Chl-a) = 0.76 log p – 0.259 (Rast and Lee (1978)
- Log (Chl-a) = 0.807 log p – 0.194 (Bartsch and Gakstatter, 1978)



Chlorophill-a versus Secchi Depth

- Log (SD) = -0.473 log (Chl-a) + 0.803 (Rast and Lee (1978)



Example

The annual loadings for a lake with 28 km^2 surface area and an average depth of 13 meters are given in the Table below. Estimate the trophic level of the lake using the Vollenweider concepts.

Source	P (kg/yr)	Flow (m^3 yr^{-1})
Urban sewage	8,000	1.0 x 10^6
Urban runoff	4,000	7.0 x 10^6
Rural runoff	26,000	10.0 x 10^6
Precipitation	1,000	21.0 x 10^6
Ground water	300	47.0 x 10^6
Evaporation	-	-8.0 x 10^6
Total	23,300	78.0

Example: solution

Flushing rate, ρ = Q/V:

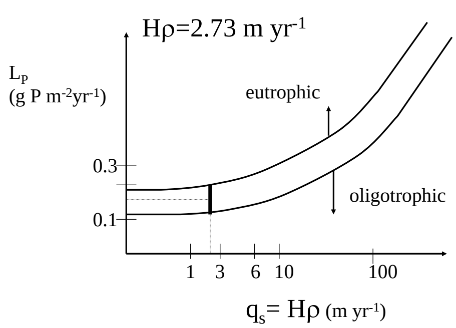
ρ = Annual flow/Lake volume =

$$\rho = \frac{78{,}000{,}000(m^3 yr^{-1})}{28(km^2)x13(m)x10^6(m^2 km^{-2})} = 0.21 yr^{-1}$$

Annual phosphorus loading, L_P:

$$L_p = \frac{23{,}000(kg \cdot yr^{-1})x10^3(g \cdot kg)}{28(km^2)x10^6(m^2 km^{-2})} = 0.83 g \cdot m^2 yr^{-1}$$

Example: solution



Example: solution

Overflow rate: q_s=Hρ=13x0.21=2.73 m yr^{-1}

Residence time: τ_w = 1/ρ = 1/0.21 = 4.78 yr

Applying $p = \frac{L_p}{q_s(1+\sqrt{\tau_w})}$: p = 95.4 μg L^{-1}

Trophic Status of Lakes

Water Quality	Oligotrophic	Mesotrophic	Eutrophic
Total P (μg/L)	<10	10-20	>20
Chlorophyll-a (μg/L)	<4	4-10	>10
Secchi depth (m)	>4	2-4	<2

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Example: solution

- Log (Chl-a) = 0.76 log p – 0.259 (Rast and Lee (1978): Chl-a = 17.6 μg L^{-1}
- Log (SD) = -0.473 log (Chl-a) + 0.803 (Rast and Lee (1978): SD = 1.6 m

Water Quality	Oligotrophic	Mesotrophic	Eutrophic
Chlorophyll-a (μg/L)	<4	4-10	>10
Secchi depth (m)	>4	2-4	<2

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Trophic Index by Carlson

- Trophic Index (TSI) developed for TP, Chlorophyll-a, and Secchi Depth:

$$TSI(TP) = 10(6 - \frac{48/TP}{\ln 2})$$

$$TSI(Chl) = 10(6 - \frac{2.04 - 0.68 \ln Chl}{\ln 2})$$

$$TSI(SD) = 10(6 - \frac{\ln SD}{\ln 2})$$

Oligotrophic: <35

Mezotrophic: 35-45

Eutrophic: >45

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Example: Tropic Index

- TP = 95.4 μg L^{-1} → TSI = 52.7
- Chl-a = 17.6 μg L^{-1} → TSI = 67.7
- SD = 1.6 m → TSI = 53.2

- Lake is eutrophic

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